

Computing Activity Chain Based Trip Distribution Models

Heinz Spiess *

March 1993

Revised April 1995
Revised November 1996

Abstract

In this note we look at a logit type choice model for simultaneously choosing the $N - 1$ intermediate destinations in an N -legged activity chain. We reformulate the model in terms of matrix products, and show how the demand matrices corresponding to the individual legs of the activity chain can be computed efficiently. An extension of the model for simultaneous destination and mode choice is also discussed, as well as the special case of mixed mode trip chains.

*EMME/2 Support Center, Haldenstrasse 16, CH-2558 Aegerten, Switzerland

1 Introduction

Traditional trip distribution models typically consider only trips to a single destination for the purpose of carrying out a single activity (such as work, shopping, leisure, ...). While this simple model of a trip represents quite well a large fraction of the trips that are subject of travel demand models, many “real life” trips combine more than one activity carried out at more than one destination (such as stopping at the shopping center on the way back from work). For this purpose, trip distribution models based on activity chains have been developed and used extensively in the past (see e.g. Adler and Ben-Akiva 1979, Clarke et al. 1981, Pas 1984, ...). Most of these models are stated in the form of a multinomial logit model, in which each possible trip chain constitutes a possible choice.

In the practical implementation of these models, however, the underlying simultaneous choice model is often approximated by using a sequential choice model, in which the decision on the n -th intermediate destinations is dependent only on the $n - 1$ previous destinations in the activity chain, but not on the destinations still ahead. It is well known (Domencich and McFadden, 1975) that under certain conditions, simultaneous logit choice models can be broken into a series of sequential sub-models. This is basically the case when the individual decisions are mutually independent. In the case of a trip chain, however, this mutual independence is clearly not given, since it is broken by the fact that the trip chain always has to end at the very same point it started out. Also, as Takahashi (1988) points out, there is empirical evidence that the choice of a destination depends substantially on the utility of possible later destinations on the same trip chain.

Nevertheless, in most practical implementations of trip chaining models, the decision making process is in some form sequentialized, in spite of the fact that this introduces approximations to the model computations.

In the following sections of this note, we will show how trip distribution models based on trip chaining can be implemented keeping their full simultaneous nature, without need for any sequentialization of the decision taking process.

2 The Three-Leg Activity Chain

Let us first look at the special case of an three-legged activity chain. Starting at the origin p , the first leg of the trip goes to the intermediate destination q , where the first activity takes place; then continues to the intermediate destination r where the second activity is carried out, and finally returns back to the origin p .

Note that we shall assume that all zone indices (such as p , q and r) always imply a zone of the same underlying zone subdivision.

The logit model for distributing the total production of trip chains starting at origin

p , T_p , to all possible pairs of intermediate destinations q and r is given by the formula

$$g_{pqrp} = t_p \frac{e^{u_{pq}+v_q+u_{qr}+w_r+u_{rp}}}{\sum_{q'r'} e^{u_{pq'}+v_{q'}+u_{q'r'}+w_{r'}+u_{r'p}}} \quad (1)$$

where u_{pq} , u_{rq} and u_{qp} represent the utilities (or rather the negatively weighted disutilities) associated with travelling on the three legs of the trip chain (pq) , (qr) and (rp) . The utility associated with carrying out the first activity at destination q is v_q and the utility of carrying out the second activity at destination r is w_r . t_p , finally, represents the total production of trips at origin p . Note that, without loss of generality, we can assume the utilities u , v and w to represent the weighted sum of all pertinent variables.

Since the exponential of the sum of the various utilities can be reformulated as the product of the exponentials of each utility, it is useful to restate in terms of the transformed variables $c_{pq} = e^{u_{pq}}$, $a_q = e^{v_q}$ and $b_r = e^{w_r}$, which yields

$$g_{pqrp} = t_p \frac{c_{pq}a_q c_{qr}b_r c_{rp}}{\sum_{q'r'} c_{pq'}a_{q'} c_{q'r'}b_{r'} c_{r'p}}. \quad (2)$$

The above formulation shows well that the the logit model with its additive character of the utility is indeed a multiplicative model in which the activities to chose from are represented by factors. Thus, it is to be noted that the discussion which follows is not limited to logit models, but is equally applicable to any model of the form (2).

The denominator of (2), which represents to total weight of all possible choices of intermediate destinations for trip chains starting at p , will be denoted by s_p

$$s_p = \sum_{qr} c_{pq}a_q c_{qr}b_r c_{rp} = \sum_q c_{pq}a_q \sum_r c_{qr}b_r c_{rp}. \quad (3)$$

While the basic result of model (2) are the number of trips (g_{pqrp}) for each possible chain $pqrp$, what for practical purposes is needed are the three demand matrices that correspond to the travel on each of the three legs of the trip chain, i.e.

$$g_{pq}^1 = \frac{t_p}{s_p} c_{pq}a_q \sum_r c_{qr}b_r c_{rp}, \quad (4)$$

$$g_{qr}^2 = a_q c_{qr}b_r \sum_p c_{rp} \frac{t_p}{s_p} c_{pq}, \quad (5)$$

$$g_{rp}^3 = b_r c_{rp} \frac{t_p}{s_p} \sum_q c_{pq}a_q c_{qr}. \quad (6)$$

It is interesting to note that the above relationships can also be formulated using matrix notation. For this purpose, we introduce the following matrices: C is the conductivity matrix c_{pq} ; A is the diagonal matrix containing attractivities a_q ; B is the diagonal

matrix containing attractivities b_p ; and finally T is the diagonal matrix containing total productions t_p .

Using matrix notation

$$S = CACBC \quad (7)$$

corresponds to the matrix of the total weights for all 3-leg trip chains starting at zone p and ending at zone q . The diagonal elements of S , therefore, correspond exactly to s_p defined in (3)

For convenience, we introduce the diagonal matrix containing the normalized trip generation rates as

$$\bar{T} = T \text{diag}(S)^{-1}. \quad (8)$$

For the demand matrices that correspond to the 3 legs of the trip chain rewriting of equations (4) - (6) in matrix notation yields

$$G_1 = (\bar{T}C) \otimes (ACBC)', \quad (9)$$

$$G_2 = (AC) \otimes (BC\bar{T}C)', \quad (10)$$

$$G_3 = (BC) \otimes (\bar{T}CAC)', \quad (11)$$

where the operator \otimes indicates element by element multiplication of the corresponding matrix operands, i.e. $Z = X \otimes Y \leftrightarrow z_{ij} = x_{ij}y_{ij}$.

This model for a 3-legged activity chain has been implemented in a macro for the EMME/2 transportation planning system. The macro is called TCHAIN3 and is available on the Web site of the EMME/2 Support Center at <http://www.spiess.ch/emme2>.

3 Multi-Leg Activity Chains

Having dealt explicitly with the special case of 3-leg trip chains, it is now relatively straight forward to extend the results to the general case of trip chains having N legs and $N-1$ intermediate destinations.

We shall denote A_1, A_2, \dots, A_{N-1} the diagonal matrices containing the attractivities of the zones for activities of the type n , $n = 1, \dots, N - 1$.

The total weights of all possible N -leg trip chains between any pair of zones is given by

$$S = CA_1CA_2C \cdots A_{N-1}C. \quad (12)$$

Again, it is convenient to introduce the diagonal matrix containing the normalized trip generation rates, but for the sake the symmetry in the notation, we shall call it A_0

$$A_0 = T \text{diag}(S)^{-1}. \quad (13)$$

Assuming that all indices of the matrix A are to be taken modulo N (i.e. $A_i = A_{(i \bmod N)}$), the demand matrix corresponding the the n -th trip leg, $n = 1, \dots, N$ can be written as

$$G_n = (A_{n-1}C) \otimes (A_n C A_{n+1} \cdots A_{n+N-2} C)'. \quad (14)$$

4 Combined Mode and Destination Choice

It is easily possible to generalize this type of distribution model to simultaneously include mode and destination choice. For the purpose, we assume that M travel modes are available to the traveller. For each mode m , a conductivity matrix C_m is constructed from the mode specific travel cost matrix. In addition, each for each origin p and each mode m , the variable f_{pm} represents the attractivity bias of mode m to the travellers from origin p (i.e. the combined effect of the mode dependent socio-economic variables at origin p). Let F_m denote the diagonal matrices containing these bias variables for mode m .

We shall denote A_{nm} the diagonal matrices containing the attractivities of the zones for activities of the type n and travel by mode n . Note that these attractivities can vary with the mode used (e.g. parking availability of cost), but they don't have to necessarily.

The total weights of all possible N -leg trip chains of all modes between any pair of zones is given by

$$S = \sum_m F_m C_m A_{1m} C_m A_{2m} C_m \cdots A_{(N-1)m} C_m. \quad (15)$$

Again, it is convenient to introduce the diagonal matrices containing the normalized trip generation rates for mode m as

$$A_{0m} = T \text{diag}(S)^{-1} F_m. \quad (16)$$

Using again the modulo N convention for the first subscript of A , the demand matrix corresponding the the n -th trip leg, $n = 1, \dots, N$ can be expressed as

$$G_{nm} = (A_{(n-1)m} C_m) \otimes (A_{nm} C_m A_{(n+1)m} C_m \cdots A_{(n+N-2)m} C_m)'. \quad (17)$$

5 Mixed-Mode Trip Chains

The type of analysis presented in the previous sections can of course also be applied to trip chains in which more the one mode is used within the same trip chain. A typical example for this category of problems is the choice of the parking site for park+ride travellers, is is briefly outlined in this section.

We assume the following logit model for choosing between parking sites:

$$g_{pkq} = g_{pq}^{\text{PR}} \frac{e^{u_{pk}^{\text{AU}} + v_k + u_{kq}^{\text{TR}}}}{\sum_{k'} e^{u_{pk'}^{\text{AU}} + v_{k'} + u_{k'q}^{\text{TR}}}}, \quad (18)$$

where g_{pkq} corresponds to the number of trips between p and q choosing parking site k according to the mode specific utilities u_{pk}^{AU} (auto mode) and u_{kq}^{TR} (transit mode) and the attractiveness variable of parking at zone k , v_k . Using the transformed variables $c_{pq}^{\text{AU}} = e^{u_{pq}^{\text{AU}}}$, $c_{pq}^{\text{TR}} = e^{u_{pq}^{\text{TR}}}$ and $a_q = e^{v_q}$, we obtain

$$g_{pkq} = g_{pq}^{\text{PR}} \frac{c_{pk}^{\text{AU}} a_k c_{kq}^{\text{TR}}}{\sum_{k'} c_{pk'}^{\text{AU}} a_{k'} c_{k'q}^{\text{TR}}}. \quad (19)$$

In matrix notation, the denominator can be written as

$$S = C_{\text{AU}} A C_{\text{TR}} \quad (20)$$

and the matrix \bar{T} is defined as

$$\bar{T} = G_{\text{PR}} \oslash S, \quad (21)$$

where the operator \oslash is used to indicate element-by-element division. The intermediate auto and transit demand matrices resulting from the above model can now be written in matrix notation as

$$G_{\text{AU}} = (C_{\text{AU}} A) \otimes (\bar{T} C'_{\text{TR}}), \quad (22)$$

$$G_{\text{TR}} = (A C_{\text{TR}}) \otimes (C'_{\text{AU}} \bar{T}). \quad (23)$$

The average auto and transit impedances for the park+ride trips from p to q , \bar{u}_{pq}^{AU} and \bar{u}_{pq}^{TR} , can be computed as

$$\bar{u}_{pq}^{\text{AU}} = \frac{\sum_k g_{pkq} u_{pk}^{\text{AU}}}{g_{pq}^{\text{PR}}} = \frac{\sum_k u_{pk}^{\text{AU}} c_{pk}^{\text{AU}} a_k c_{kq}^{\text{TR}}}{s_{pq}}, \quad (24)$$

$$\bar{u}_{pq}^{\text{TR}} = \frac{\sum_k g_{pkq} u_{kq}^{\text{TR}}}{g_{pq}^{\text{PR}}} = \frac{\sum_k c_{pk}^{\text{AU}} a_k c_{kq}^{\text{TR}} u_{kq}^{\text{TR}}}{s_{pq}}, \quad (25)$$

whereas the average parking cost paid by park+ride trip from p to q is

$$\bar{v}_{pq} = \frac{\sum_k g_{pkq} v_k}{g_{pq}^{\text{PR}}} = \frac{\sum_k c_{pk}^{\text{AU}} a_k v_k c_{kq}^{\text{TR}}}{s_{pq}}. \quad (26)$$

In matrix notation, the park+ride impedance and cost matrices can be written as

$$\bar{U}_{\text{AU}} = ((U_{\text{AU}} \otimes C_{\text{AU}}) A C_{\text{TR}}) \oslash S, \quad (27)$$

$$\bar{U}_{\text{TR}} = (C_{\text{AU}} A (C_{\text{TR}} \otimes U_{\text{TR}})) \oslash S, \quad (28)$$

$$\bar{V} = (C_{\text{AU}} (A \otimes V) C_{\text{TR}}) \oslash S. \quad (29)$$

Note that the above formulae are not limited to compute the impedances and parking costs of the park+ride trip, but they can also be applied to any other auto or transit attribute, such as distance or fare, by simply substituting the matrix U by the corresponding attribute matrix.

The computations of the first and second leg intermediate matrices for mixed mode trips according to (22) and (23) have been implemented in a macro for the EMME/2 transportation planning system. The macro is called **MIMOLEG** and is available on the Web site of the EMME/2 Support Center at <http://www.spiess.ch/emme2>.

Note that the model presented in this note do not consider any capacities limitations at parking lots. For a detailed discussion of modeling mixed mode trips with explicit capacities at intermediate destinations see Spiess (1996).

6 Conclusions

We have shown that logit type distribution models that are based on activity chains can be computed directly without any need to approximate the behavioral assumptions through the introduction of a sequential decision model. Thus the traveller decides for a trip chain in its entirety, i.e. all intermediate destinations (as well as the mode, in the case of a combined model) are chosen simultaneously.

We showed that these models can be reformulated as simple matrix expressions, involving only simple operations such as algebraic matrix multiplications and element-by-element matrix multiplications. This also means that they can be efficiently implemented in any transportation planning software packages which includes matrix calculation facilities which include these two types of operations. The EMME/2 transportation planning system (Spiess 1984, INRO 1992) includes very flexible matrix manipulation tools. These have been used to implement activity chain based distribution models by directly applying the formulae developed in the preceding section. Macros have been written to automate the computations. This shows that direct implementation of activity chain based trip distribution models are not just theoretically possible, but also practically feasible, even for large scale applications.

References

- [1] Adler T. and Ben-Akiva M. (1979), *A Theoretical and Empirical Model of Trip Chaining Behavior*. Transportation Research 13B, 243-257.
- [2] Clarke M.I., Dix M.C., Jones P.M. and Heggie I.G. (1981), *Some Recent Developments in Activity-Travel Analysis*. Transportation Research Record 794, 1-8.

- [3] Domencich T. and McFadden D. (1975), *Urban Travel Demand: A Behavioral Analysis*. North-Holland, Amsterdam.
- [4] INRO Consultants Inc. (1992), *EMME/2 User's Manual*.
- [5] Pas E.I. (1984), *The Effect of Selected Sociodemographic Characteristics on Daily Travel-Activity Behavior*. Environment and Planning A, 16, 571-581.
- [6] Spiess H. (1984). *Contributions à la théorie et aux outils de planification de réseaux de transport urbain*. Ph.D. thesis, Département d'informatique et de recherche opérationnelle, Centre de recherche sur les transports, Université de Montréal, Publication 382.
- [7] Spiess H. (1996). *A Logit Parking Choice Model with Explicit Capacities*. EMME/2 Support Center, CH-2558 Aegerten.
- [8] Takahashi S. (1988) *Multiple Stops and Retail Choice: Their Intraurban Variations*. Ph.D thesis, University of Kansas.